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EQUILIBRIUM OF A SLOPE WITH A TECTONIC CRACK

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Equilibrium of an elastic half-plane with a rectilinear crack reaching the half-plane free boundary at an arbitrary angle is considered as a plane problem of the theory of elasticity. It is assumed that known compressive stresses are applied at considerable distance from the crack forcing the opposite boundaries of the crack to contact each other. Interaction between the crack boundaries are defined by the law of dry friction with cohesion. Mathematically this problem is analogous to that of a tectonic crack filled with a low-strength medium. First, the problem is stated and fundamental relationships are presented. The Wiener-Hopf equation of the considered problem is derived with the use of Mellin transform and Jones method. The exact analytical solution of the Wiener-Hopf equations is then

obtained and the stress intensity coefficient at the tip of the crack is determined. The obtained solution is used for investigating the geophysical problem of stability of a slope with a tectonic crack.

1. Statement of the problem and fundamental relationships.

Let us consider the deformation of a homogeneous isotropic elastic half-space under conditions of plane strain. In the Oxy-plane of rectangular Cartesian coordinates xy the half-space occupies the half-plane $x > 0$. A rectilinear crack of length l reaches at angle α to the x-axis the half-plane boundary which is free of external load (Fig. 1).

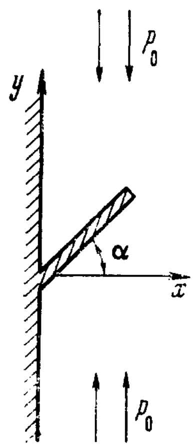

Fig. 1

Below we shall use polar coordinates $r\theta$ with their center at the origin of Cartesian coordinates.

We assume that compressive stresses acting at infinity force the opposite sides of the crack together, and that the interaction between these is defined by the law of Coulomb dry friction with adhesion. It is also possible to consider this problem as one of a tectonic crack with a low strength filler, a problem that is mathematically analogous to the considered one (more about this in Sect. 5 below).

The fundamental relationships (the equation of equilibrium, the condition of strain compatibility, and Hooke's law) are

$$r\frac{\partial\sigma_r}{\partial r}+\frac{\partial\tau_{r\theta}}{\partial\theta}+\sigma_r-\sigma_\theta=0 \qquad (1.1)$$

$$\frac{\partial\sigma_\theta}{\partial\theta}+r\frac{\partial\tau_{r\theta}}{\partial r}+2\tau_{r\theta}=0$$

$$\triangle(\sigma_r+\sigma_\theta)=0 \quad \left(\triangle=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right) \qquad (1.2)$$

$$\frac{\partial u_r}{\partial r}=\frac{1+\nu}{E}[(1-\nu)\sigma_r-\nu\sigma_\theta] \qquad (1.3)$$

$$\frac{u_r}{r}+\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}=\frac{1+\nu}{E}[(1-\nu)\sigma_\theta-\nu\sigma_r]$$

$$\frac{1}{r}\frac{\partial u_r}{\partial\theta}+\frac{\partial u_\theta}{\partial r}-\frac{u_\theta}{r}=2\frac{1+\nu}{E}\tau_{r\theta}$$

The length of the crack can be taken, without loss of generality, as the characteristic unit of length.

We formulate the boundary conditions of the problem as follows:

$$\text{for}\quad \theta=\pm\pi/2 \quad \sigma=\tau_{r\theta}=0 \qquad (1.4)$$

$$\text{for}\quad \theta=\alpha \quad [\sigma_\theta]=[\tau_{r\theta}]=0,\quad [u_\theta]=0 \qquad (1.5)$$

$$\text{for}\quad \theta=\alpha,\quad 0<r<1 \quad \tau_{r\theta}=k-\sigma_\theta\,\mathrm{tg}\,\rho+f(r) \qquad (\sigma_\theta<0) \quad (1.6)$$

$$\text{for}\quad \theta=\alpha,\quad 1<r<\infty \quad [\sigma_r]=0 \qquad (1.7)$$

where k is the adhesion coefficient, ρ is the angle of dry friction between the crack sides, $f(r)$ is a specified function, and brackets denote a jump of the enclosed quantity (e.g., $[\sigma_\theta]=\sigma_\theta|_{\theta=\alpha+0}-\sigma_\theta|_{\theta=\alpha-0}$). For $0<r<1$ conditions (1.5) mean in the physical sense that there are no external loads on the crack and that its sides are in close contact all along its length. Function $f(r)$ is determined from the external problem with the use of the superposition principle so as to have the condition

$$\text{for} \quad r \to \infty, \quad \sigma_\theta \to 0, \quad \tau_{r\theta} \to 0, \quad \sigma_r \to 0 \tag{1.8}$$

satisfied at infinity. For example, if a constant compressive stress $\sigma_y = -p_0$ is present at infinity and all remaining stresses are zero (Fig. 1), function $f(r)$ is equal to the following constant quantity:

$$f(r) = -\frac{1}{2} p_0 \frac{\sin\rho + \sin(\rho + 2\alpha)}{\cos\rho} \tag{1.9}$$

It is equal $(\tau_{r\theta} + \sigma_\theta \operatorname{tg}\rho)$ taken with the minus sign, which corresponds to a solid body without a crack along the line $\theta = \alpha$. Stresses and strains in the problem represented in Fig. 1 are, evidently, equal to the stresses and strains obtained in the solution of the boundary value problem (1.1) — (1.8) plus the related stresses and strains in the same solid body without a crack subjected to compressive stress $\sigma_y = -p_0$. The definition of function $f(r)$ in the case of a heavy slope at an arbitrary angle subjected to variable external load is given in Sect. 5. The problem of equilibrium of a brittle body with cracks superposed along one and the same straight line was previously considered in a similar physical formulation [1].

2. The Wiener-Hopf equation. Let us apply the Mellin transform [2]

$$f^*(p) = \int_0^\infty f(r) r^p dr \qquad (p \text{ is a complex parameter}) \tag{2.1}$$

to the equilibrium equations (1.1). We obtain

$$\tau_{r\theta}^* = \frac{1}{p-1}\frac{d\sigma_\theta^*}{d\theta}, \quad p\sigma_r^* = \frac{1}{p-1}\frac{d^2\sigma_\theta^*}{d\theta^2} - \sigma_\theta^* \tag{2.2}$$

Physical considerations indicate that stresses in this problem are limited for $r \to 0$ and for $r \to \infty$ they conform to $O(r^{-2})$. Hence their Mellin transforms are analytic functions of the complex variable p in the strip $-1 < \operatorname{Re} p < 1$. Substituting σ_r^* into the transformed equation (1.2), we obtain

$$\frac{d^4\sigma_\theta^*}{d\theta^4} + [(p+1)^2 + (p-1)^2]\frac{d^2\sigma_\theta^*}{d\theta^2} + (p+1)^2(p-1)^2\sigma_\theta^* = 0 \tag{2.3}$$

We represent the solution of this equation thus:

$$\sigma_\theta^* = \begin{cases} \sigma_\theta^{*+}, & -\pi/2 \leqslant \theta < \alpha \\ \sigma_\theta^{*-}, & \alpha < \theta \leqslant \pi/2 \end{cases} \tag{2.4}$$

$$\sigma_\theta^{*\pm} = A_1^\pm \sin(p+1)\left(\theta \pm \frac{\pi}{2}\right) + A_2^\pm \sin(p-1)\left(\theta \pm \frac{\pi}{2}\right) + A_3^\pm \cos(p+1)\left(\theta \pm \frac{\pi}{2}\right) + A_4^\pm \cos(p-1)\left(\theta \pm \frac{\pi}{2}\right) \tag{2.5}$$

where $A_1^\pm$, $A_2^\pm$, $A_3^\pm$ and $A_4^\pm$ are unknown functions of parameter p that are to be determined from boundary conditions. Any seven of these can be expressed in terms of a single unknown function by using seven "through" boundary conditions (1.4) and (1.5) transformed with respect to r. By (1.3) we have

$$\frac{\partial^2 u_\theta}{\partial r^2} = \frac{1+\nu}{E}\left(2\frac{\partial \tau_{r\theta}}{\partial r} + 2\frac{\tau_{r\theta}}{r} - \frac{1-\nu}{r}\frac{\partial \sigma_r}{\partial \theta} + \frac{\nu}{r}\frac{\partial \sigma_\theta}{\partial \theta}\right)$$

and, consequently,
$$(p+1)\frac{\partial u_\theta^*}{\partial r} = \frac{1+\nu}{E}\left[2p\tau_{r\theta}^* + (1-\nu)\frac{d\sigma_r^*}{d\theta} - \nu\frac{d\sigma_\theta^*}{d\theta}\right] \qquad (2.6)$$

From the transformed boundary conditions (1.4) and (1.5) with the use of (2.2) and (2.6) we obtain
$$A_3^- + A_4^- = 0, \quad A_3^+ + A_4^+ = 0 \qquad (2.7)$$
$$A_1^+(p+1)\cos(p+1)\left(\alpha+\frac{\pi}{2}\right) + A_2^+(p-1)\cos(p-1)\times$$
$$\left(\alpha+\frac{\pi}{2}\right) - A_3^+(p+1)\sin(p+1)\left(\alpha+\frac{\pi}{2}\right) -$$
$$A_4^+(p-1)\sin(p-1)\left(\alpha+\frac{\pi}{2}\right) = A_1^-(p+1)\cos(p+1)\times$$
$$\left(\alpha-\frac{\pi}{2}\right) + A_2^-(p-1)\cos(p-1)\left(\alpha-\frac{\pi}{2}\right) -$$
$$A_3^-(p+1)\sin(p+1)\left(\alpha-\frac{\pi}{2}\right) - A_4^-(p-1)\sin(p-1)\times$$
$$\left(\alpha-\frac{\pi}{2}\right) - A_1^+(p+1)^3\cos(p+1)\left(\alpha+\frac{\pi}{2}\right) -$$
$$A_2^+(p-1)^3\cos(p-1)\left(\alpha+\frac{\pi}{2}\right) + A_3^+(p+1)^3\sin(p+1)\times$$
$$\left(\alpha+\frac{\pi}{2}\right) + A_4^+(p-1)^3\sin(p-1)\left(\alpha+\frac{\pi}{2}\right) = -A_1^-(p+1)^3\times$$
$$\cos(p+1)\left(\alpha-\frac{\pi}{2}\right) - A_2^-(p-1)^3\cos(p-1)\left(\alpha-\frac{\pi}{2}\right) +$$
$$A_3^-(p+1)^3\sin(p+1)\left(\alpha-\frac{\pi}{2}\right) + A_4^-(p-1)^3\sin(p-1)\left(\alpha-\frac{\pi}{2}\right)$$
$$A_1^-(p+1) + A_2^-(p-1) = 0, \quad A_1^+(p+1) + A_2^+(p-1) = 0$$
$$A_1^+\sin(p+1)\left(\alpha+\frac{\pi}{2}\right) + A_2^+\sin(p-1)\left(\alpha+\frac{\pi}{2}\right) +$$
$$A_3^+\cos(p+1)\left(\alpha+\frac{\pi}{2}\right) + A_4^+\cos(p-1)\left(\alpha+\frac{\pi}{2}\right) =$$
$$A_1^-\sin(p+1)\left(\alpha-\frac{\pi}{2}\right) + A_2^-\sin(p-1)\left(\alpha-\frac{\pi}{2}\right) +$$
$$A_3^-\cos(p+1)\left(\alpha-\frac{\pi}{2}\right) + A_4^-\cos(p-1)\left(\alpha-\frac{\pi}{2}\right)$$

We write the solution of this system of equations in the form
$$A_1^\pm = \Delta_1^\pm(p)D(p), \quad A_2^\pm = -\frac{p+1}{p-1}\Delta_1^\pm(p)D(p) \qquad (2.8)$$
$$A_3^\pm = \Delta_2^\pm(p)D(p), \quad A_4^\pm = -\Delta_2^\pm(p)D(p)$$

where ($D(p)$ is an unknown function of p and
$$\Delta_1^\pm = \pm\begin{vmatrix} c_1^- & a_1^\mp & c_1^+ \\ c_2^- & a_2^\mp & c_2^+ \\ c_3^- & a_3^\mp & c_3^+ \end{vmatrix}, \quad \Delta_2^\pm = \begin{vmatrix} a_1^+ & a_1^- & c_1^\mp \\ a_2^+ & a_2^- & c_2^\mp \\ a_3^+ & a_3^- & c_3^\mp \end{vmatrix} \qquad (2.9)$$
$$a_1^\pm = (p\mp1)\sin(p\pm1)\left(\alpha\pm\frac{\pi}{2}\right) - (p\pm1)\sin(p\mp1)\left(\alpha\pm\frac{\pi}{2}\right)$$

$$a_2^{\pm} = -2(p+1)\cos\alpha \sin p\left(\alpha \pm \frac{\pi}{2}\right)$$

$$a_3^{\pm} = (p+1)\left[(p\mp 1)^2 \cos(p\mp 1)\left(\alpha \pm \frac{\pi}{2}\right) - (p\pm 1)^2\cos(p\pm 1)\left(\alpha\pm\frac{\pi}{2}\right)\right]$$

$$c_1^{\pm} = \mp 2(p-1)\cos\alpha\sin p\left(\alpha\pm\frac{\pi}{2}\right)$$

$$c_2^{\pm} = (p-1)\sin(p-1)\left(\alpha\pm\frac{\pi}{2}\right) - (p+1)\sin(p+1)\left(\alpha\pm\frac{\pi}{2}\right)$$

$$c_3^{\pm} = (p+1)^3\sin(p+1)\left(\alpha\pm\frac{\pi}{2}\right) - (p-1)^3\sin(p-1)\left(\alpha\pm\frac{\pi}{2}\right)$$

The boundary conditions at displacements ($[u_\theta] = 0$ for $\theta = \alpha$ and $[u_r] = 0$ for $\theta = \alpha\, r > 1$) may be taken as automatically satisfied owing to the single-connectedness of the considered region, since the conditions $[\partial u_\theta / \partial r] = 0$ and $[\partial u_r / \partial r] = 0$ are satisfied for $\theta = \alpha$ and $\theta = \alpha r > 1$, respectively. In any case the addition of a constant dislocation jump along the line $\theta = \alpha$ does not affect the strain and stress fields.

Omitting cumbersome computations, we present the results of calculations by formulas (2.9)

$$\Delta_1^{\pm}(p) = \pm 16p(p^2-1)\cos\alpha\sin\pi p\sin p\left(\alpha\mp\frac{\pi}{2}\right) \tag{2.10}$$

$$\Delta_2^{\pm}(p) = -16p(p+1)\sin\pi p\left[\pm p\cos\alpha\cos p\left(\alpha\mp\frac{\pi}{2}\right)\mp \sin\alpha\sin p\left(\alpha\mp\frac{\pi}{2}\right)\right]$$

We introduce the following functions:

$$\Phi^-(p) = \int_0^1 [\sigma_r]|_{\theta=\alpha} r^p\,dr,\quad \Psi^+(p) = \int_1^\infty (\tau_{r\theta} + \sigma_\theta \operatorname{tg}\rho)|_{\theta=\alpha} r^p\,dr \tag{2.11}$$

Functions $\Phi^-(p)$ and $\Psi^+(p)$ are obviously analytic in the half-planes $\operatorname{Re} p > -1$ and $\operatorname{Re} p < 1$, respectively.

Boundary conditions (1.7) and (1.6) imply that

$$\text{for}\quad \theta=\alpha\quad [\bar\sigma_r] = \Phi^-(p) \tag{2.12}$$

$$\text{for}\quad \theta=\alpha\quad \tau^*_{r\theta} + \sigma^*_\theta \operatorname{tg}\rho = \Psi^+(p) + F(p) \tag{2.13}$$

$$F(p) = \int_0^1 [k + f(r)]\, r^p\,dr \tag{2.14}$$

Using formulas (2.2), (2.4), (2.5) and (2.8) with conditions (2.12) and (2.13), we obtain

$$p(p-1)\,\Phi^-(p) = C_1(p)D(p) \tag{2.15}$$

$$(p-1)\,\{\Psi^+(p) + F(p)\} = D(p)\,\{2C_2(p) + C_3(p)\operatorname{tg}\rho\} \tag{2.16}$$

where

$$C_1(p) = \Delta_1^+(p)(p+1)^2\sin(p+1)\left(\alpha+\frac{\pi}{2}\right) - \Delta_1^+(p)(p^2-1)\sin(p-1)\left(\alpha+\frac{\pi}{2}\right) + \Delta_2^+(p)(p+1)^2\times \tag{2.17}$$

$$\cos(p+1)\left(\alpha+\frac{\pi}{2}\right)-\Delta_2^+(p)(p-1)^2\cos(p-1)\left(\alpha+\frac{\pi}{2}\right)-$$
$$\Delta_1^-(p)(p+1)^2\sin(p+1)\left(\alpha-\frac{\pi}{2}\right)+\Delta_1^-(p)(p^2-1)\times$$
$$\sin(p-1)\left(\alpha-\frac{\pi}{2}\right)-\Delta_2^-(p)(p+1)^2\cos(p+1)\times$$
$$\left(\alpha-\frac{\pi}{2}\right)+\Delta_2^-(p)(p-1)^2\cos(p-1)\left(\alpha-\frac{\pi}{2}\right)$$
$$C_2(p)=-\Delta_1^+(p)(p+1)\cos\alpha\sin p\left(\alpha+\frac{\pi}{2}\right)-$$
$$\Delta_2^+(p)\,p\cos\alpha\cos p\left(\alpha+\frac{\pi}{2}\right)+\Delta_2^+(p)\sin\alpha\sin p\left(\alpha+\frac{\pi}{2}\right)$$
$$C_3(p)=\Delta_1^+(p)(p-1)\sin(p+1)\left(\alpha+\frac{\pi}{2}\right)-$$
$$\Delta_1^+(p)(p+1)\sin(p-1)\left(\alpha+\frac{\pi}{2}\right)-2\Delta_2^+(p)\times$$
$$(p-1)\cos\alpha\sin p\left(\alpha+\frac{\pi}{2}\right)$$

Omitting intermediate computations, we present the results of transformation by formulas (2.17) and (2.10)

$$C_1(p)=64p^2(p+1)\sin^2\pi p \tag{2.18}$$
$$C_2(p)=8p(p+1)\sin\pi p\,(2p^2\cos^2\alpha\cos 2p\alpha-$$
$$p\sin 2\alpha\sin 2p\alpha+\cos\pi p-\cos 2p\alpha)$$
$$C_3(p)=32p^2(p^2-1)\cos^2\alpha\sin\pi p\sin 2p\alpha$$

Eliminating function $D(p)$ from formulas (2.15) and (2.16) we obtain the following functional Wiener-Hopf equation:

$$\Psi^+(p)+F(p)={}^1/_4\operatorname{ctg}\pi p\,G(p)\,\Phi^-(p) \tag{2.19}$$

$$G(p)=1+\frac{\cos 2p\alpha}{\cos\pi p}\{2p^2\cos^2\alpha-1+p\operatorname{tg}2p\alpha[2(p-1)\operatorname{tg}\rho\cos^2\alpha-\sin 2\alpha]\} \tag{2.20}$$

3. Solution of the boundary value problem. It can be readily shown that function $G(p)$ in the functional equation (2.19) has the following properties:

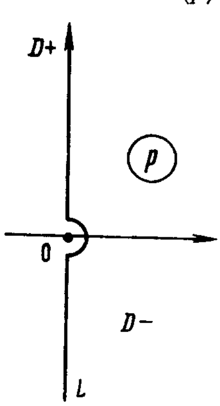

Fig. 2

a) it is meromorphic and all poles which lie at points $p=\pm{}^1/_2\pm n$, where $n=1,2,3,\ldots$ are simple;

b) for $\rho\neq 0$ it has no poles or zeros anywhere along the imaginary axis, with the exception of point $p=0$, where it has the second order zero

$$G(p)=p^2\left[2\alpha^2-\frac{\pi^2}{2}+2\cos^2\alpha-\right. \tag{3.1}$$
$$\left.4\alpha\cos\alpha(\operatorname{tg}\rho\cos\alpha+\sin\alpha)\right]+O(p^3)\ (p\to 0)$$

c) for $p\to\infty$ it tends to unity along the imaginary axis in virtue of the inequality $\alpha<\pi/2$.

Let us consider contour L in plane p consisting of the imaginary axis (with the exception of a small segment symmetric about the coordinate origin) and of the right-hand small radius semicircle with its center at the coordinate origin (Fig. 2). The direction of passing the contour L coincides with that of the imaginary axis. Regions lying to the left and right of contour L are denoted by D_+ and D_- respectively. Along the contour L function $G(p)$ has obviously nowhere either poles or zeros, and can be represented in the form

$$G(p) = G^+(p) \,/\, G^-(p) \qquad (p \in L) \tag{3.2}$$

where $G^+(p)$ and $G^-(p)$ are analytic functions that have no zeros in regions D_+ and D_-, respectively.

Functions $G^+(p)$ and $G^-(p)$ can be defined as follows:

$$\exp \frac{1}{2\pi i} \int_L \frac{\ln G(t)\, dt}{t-p} = \begin{cases} G^+(p), & p \in D_+ \\ G^-(p), & p \in D_- \end{cases} \tag{3.3}$$

Owing to the properties of Cauchy type integrals, functions $G^+(p)$ and $G^-(p)$ satisfy all specified conditions (condition (3.2) is readily verified by passing in formulas (3.3) to limit values of functions along contour L with the use of Sokhotski's formula). Since for $|t| \to \infty$ function $\ln G(t)$ exponentially decreases along L, integral (3.3) rapidly converges.

We shall use also the following known formula (see, e.g., [3]):

$$p \operatorname{ctg} \pi p = K^+(p) K^-(p) \tag{3.4}$$

where

$$K^\pm(p) = \Gamma(1 \mp p) \,/\, \Gamma(1/2 \mp p) \tag{3.5}$$

In conformity with the properties of gamma functions, functions $K^+(p)$ and $K^-(p)$ are analytic and have no zeros for $\operatorname{Re} p < 1/2$ and $\operatorname{Re} p > -1/2$, respectively. Furthermore, according to the Stirling formula we have

$$\text{for } p \to \infty \quad K^+(p) = \sqrt{-p} + O(1) \quad (\operatorname{Re} p < 1/2) \tag{3.6}$$
$$K^-(p) = \sqrt{p} + O(1) \quad (\operatorname{Re} p > -1/2)$$

By factorizing (3.2) and (3.4) the functional equation (2.19) can be written as

$$\frac{\Psi^+(p)}{K^+(p) G^+(p)} + \frac{F(p)}{K^+(p)\, G^+(p)} = \frac{\Phi^-(p) K^-(p)}{4pG^-(p)} \qquad (p \in L) \tag{3.7}$$

Let us now use the following formula:

$$\frac{F(p)}{K^+(p)\, G^+(p)} = F^+(p) - F^-(p) \qquad (p \in L) \tag{3.8}$$

where

$$\frac{1}{2\pi i} \int_L \frac{F(t)\, dt}{K^+(t)\, G^+(t)\,(t-p)} = \begin{cases} F^+(p), & p \in D_+ \\ F^-(p), & p \in D_- \end{cases} \tag{3.9}$$

Substituting (3.8) into (3.7) we obtain

$$\frac{\Psi^+(p)}{K^+(p)\, G^+(p)} + F^+(p) = \frac{\Phi^-(p)\, K^-(p)}{4pG^-(p)} + F^-(p) \qquad (p \in L) \tag{3.10}$$

The function in the left-hand part of this equality is analytic throughout region D_+, while that in its right-hand part is analytic throughout region D_-. By the principle of analytic continuation they are equal to one and the same function that is analytic throughout the region. To determine that function, it is necessary to consider the behavior of the unknown functions $\Phi^-(p)$ and $\Psi^+(p)$ at infinity for $p \to \infty$. For this we use formulas (2.10) and the asymptotic properties of the elastic field close to the tip of the crack with contacting sides [1]. Under the conditions of the considered problem and for $\theta \to \alpha$ and $r \to 1$ we have

$$\sigma_r = -\frac{K_{II}}{\sqrt{2\pi r_0}} \sin\frac{\varphi}{2}\left(2+\cos\frac{\varphi}{2}\cos\frac{3\varphi}{2}\right) + O(1) \tag{3.11}$$

$$\sigma_\theta = \frac{K_{II}}{\sqrt{2\pi r_0}} \sin\frac{\varphi}{2}\cos\frac{\varphi}{2}\cos\frac{3\varphi}{2} + O(1) \quad (r_0 \ll 1)$$

$$\tau_{r\theta} = \frac{K_{II}}{\sqrt{2\pi r_0}} \cos\frac{\varphi}{2}\left(1-\sin\frac{\varphi}{2}\sin\frac{3\varphi}{2}\right) + O(1)$$

where r_0 is the distance from the crack tip, which is small in comparison with the crack length; φ is the angle of the radius vector (at the crack tip) to the crack continuation (line) $\theta = \alpha$, and K_{II} is the stress intensity factor. Parameter K_{II}, which is of considerable importance in fracture mechanics, remains to be determined in the course of solving the problem. In accordance with (3.11) we have

$$\text{for } \theta = \alpha,\ r = 1 - r_0 \quad (r_0 \ll 1) \quad [\sigma_r] = -\frac{4K_{II}}{\sqrt{2\pi r_0}} \tag{3.12}$$

$$\text{for } \theta = \alpha,\ r = 1 + r_0 \quad (r_0 \ll 1) \quad \tau_{r\theta} = \frac{K_{II}}{\sqrt{2\pi r_0}}, \quad \sigma_\theta = O(1)$$

Reducing integrals (2.10) to the standard form along the semi-infinite interval by using the new variable $t = \ln r$, then, applying a kind of Abel's theorem [3], from formulas (3.12) we obtain

$$\text{for } p \to \infty \quad \Phi^-(p) = -\frac{2\sqrt{2}\,K_{II}}{\sqrt{p}}, \quad \Psi^+(p) = \frac{K_{II}}{\sqrt{-2p}} \tag{3.13}$$

Consequently, on the basis of formulas (3.3), (3.6), (3.9) and (3.13) the analytic function in the right-hand part of equality (3.10) tends to vanish in D_- when $p \to \infty$. Thus by the Liouville's theorem the single analytic function is zero throughout the p-plane.

The final solution of the problem can be written as

$$\Phi^-(p) = -\frac{4pF^-(p)\,G^-(p)}{K^-(p)} \tag{3.14}$$

$$\Psi^+(p) = -F^+(p)\,K^+(p)\,G^+(p) \tag{3.15}$$

From this we determine function $D(p)$ using (2.15) or (2.16) and obtain the Mellin transform for the unknown stresses. After inversion of the transformation we obtain the stresses themselves.

4. Analysis of solution. Let us determine the coefficient of stress intensity at the tip of the crack by using the known asymptotic behavior of (3.13) and formulas (3.14), (3.3), (3.9) and (3.6) for $p \to \infty$. We obtain

$$K_{II} = -\frac{1}{\sqrt{2}\,\pi i}\int_L \frac{F(t)\,dt}{K^+(t)\,G^+(t)} \tag{4.1}$$

It is possible to reduce this double integral to a single integral in the most frequently occurring case when the external load is defined by a polynomial or a rational-fractional function. To illustrate this we present three examples.

Constant load. Let function $f(r)$ be constant, $f(r) = a = \text{const}$. (In the case of uniform compression at infinity, shown in Fig. 1, this constant is defined by formula (1.9)).

Using formula (2.14) we obtain in this case

$$F(p) = (a + k) / (p + 1) \tag{4.2}$$

Substituting this expression into formula (4.1) and computing the integral by the theory of residues, we obtain

$$K_{II} = -\frac{\sqrt{2}\,(a+k)}{K^+(-1)G^+(-1)} = -\frac{\sqrt{\pi}\,(a+k)}{\sqrt{2}\;G^+(-1)} \tag{4.3}$$

Using formulas (3.3) and (2.20) we transform the integral $G^+(-1)$ to a form convenient for computation. First we note that the integral

$$\frac{1}{2\pi i}\int \frac{\ln G(t)\,dt}{t+1}$$

taken over the small semicircle of radius ε near the coordinate origin is in conformity with expansion (3.1) of order $\varepsilon \ln \varepsilon + O(\varepsilon)$, i.e. it tends to zero when $\varepsilon \to 0$. Hence in computing $G^+(-1)$ it is possible to take the imaginary axis as the contour L The integral is then convergent (with a logarithmic singularity of the integrand at the coordinate origin). Next, we introduce along the contour L the real variable τ ($p = i\tau$) and transform the integral to the semi-infinite interval $(0, +\infty)$ noting that along the imaginary axis $\operatorname{Re} G$ is an even and $\operatorname{Im} G$ an odd function of τ. As the result we obtain

$$G^+(-1) = \exp \frac{1}{\pi}\int_0^\infty \frac{\ln|G| + \tau \arg G}{1+\tau^2}\,d\tau \tag{4.4}$$

where

$$\ln|G| = {}^1/_2 \ln\left[(\operatorname{Re} G)^2 + (\operatorname{Im} G)^2\right] \tag{4.5}$$

$$\arg G = \operatorname{arctg}\frac{\operatorname{Im} G}{\operatorname{Re} G}$$

$$\operatorname{Im} G = -2\tau^2 \operatorname{tg}\rho \cos^2\alpha \operatorname{th} 2\alpha\tau \frac{\operatorname{ch} 2\alpha\tau}{\operatorname{ch}\pi\tau}$$

$$\operatorname{Re} G = 1 - \frac{\operatorname{ch} 2\alpha\tau}{\operatorname{ch}\pi\tau}\left[1 + 2\tau^2\cos^2\alpha - \tau \operatorname{th} 2\alpha\tau\,(\sin 2\alpha + 2\operatorname{tg}\rho\cos^2\alpha)\right]$$

The expression $G^+(-1)$ is a function that depends only on α and ρ. Values of $G^+(-1) \times 10^3$ are presented in Table 1 for various ρ and α; these data were obtained by approximate integration of (4.4) by Simpson's rule on a computer. To improve the convergence it is advisable, first, to eliminate the logarithmic singularity of the integrand at zero with the use of the following identity:

$$\frac{2}{\pi}\int_0^1 \frac{\ln\tau}{1+\tau}\,d\tau = -\frac{\pi}{6} \tag{4.6}$$

Linear load. Let function $f(r)$ be linear, $f(r) = \eta_1 r$, where η_1 is some constant coefficient. Such case occurs, for instance, when the own weight of a heavy massif, or the asymptotic bending are taken into consideration.

In this case, using formula (2.14), we obtain

$$F(p) = \frac{k}{p+1} + \frac{\eta_1}{p+2} \tag{4.7}$$

Using (4. 1), (3. 3) and (3. 5) and restricting (4. 7) to the second term, we obtain

$$K_{\text{II}} = -\frac{\sqrt{2}\,\eta_1}{K^+(-2)\,G^+(-2)} = -\frac{3\sqrt{2\pi}\,\eta_1}{8G^+(-2)} \tag{4.8}$$

where

$$G^+(-2) = \exp\frac{1}{\pi}\int_0^\infty \frac{2\ln|G| + \tau\arg G}{4+\tau^2}\,d\tau \tag{4.9}$$

and the remaining expressions are given in (4. 5). Values of $G^+(-2) \times 10^3$ computed for various ρ and α by the same method as in the previous case, are presented in Table 2.

Quadratic load. Let function $f(r)$ be quadratic, $f(r) = \eta_2 r^2$, where η^2 is some constant coefficient. In this case on the basis of (2. 14) we have

$$F(p) = \frac{k}{p+1} + \frac{\eta_2}{p+3} \tag{4.10}$$

Using (4. 1), (3, 3) and (2. 20) and restricting (4. 10) to the second term we obtain

$$K_{\text{II}} = -\frac{\sqrt{2}\,\eta_2}{K^+(-3)\,G^+(-3)} = -\frac{5\eta_2\sqrt{2\pi}}{16G^+(-3)} \tag{4.11}$$

where

$$G^+(-3) = \exp\frac{1}{\pi}\int_0^\infty \frac{3\ln|G| + \tau\arg G}{9+\tau^2}\,d\tau \tag{4.12}$$

and the remaining expressions are given in (4. 5). Values of $G^+(-3) \times 10^3$ computed for various ρ and α by the same method as in previous cases, are given in Table 3.

5. Stability of a slope with a tectonic crack. The derived mathematical solution can be applied in investigations of the slip line development in metals and in cracked rocks. According to the general functional method of fracture mechanics [1] the development of slip line (or fault fissure) can take place only then (*) when the absolute value of the stress concentration factor K_{II} equals at the considered point some constant K_{IIc}. The latter (the slip ductility) is completely determined by the structure and strength of the material at the "head" of the slip line. We recall the formula $K_{\text{IIc}}^2 = 2E\gamma_* / (1 - \nu^2)$, where γ_* is the irreversible work of plastic deformation at the slip line head expended on the formation of unit area of the slip surface. We stress that the work of irreversible deformations at the already existing slip surface is not included in γ_*. We shall use this criterion for the theoretical solution of the geophysical problem of stability of a slope with a tectonic fissure.

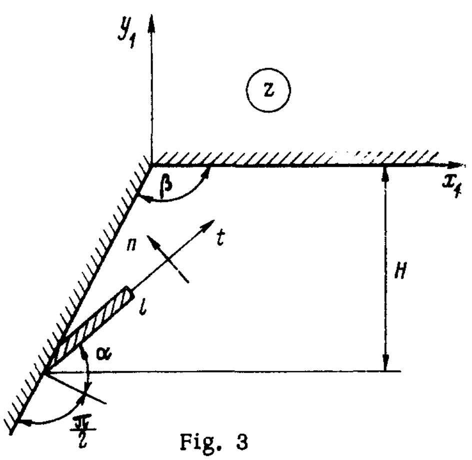

Fig. 3

Let a heavy isotropic elastic massif occupy sector $y_1 < 0$, $x_1 \operatorname{tg}\beta + y_1 < 0$ with

*) Other, more complex criteria that take into account various temporal processes, the external medium, macroplasticity, etc., are possible [1].

Table 1

ρ \ α	0°	15°	30°	45°	60°	75°	85°
0°	448	458	486	502	491	418	257
5°	448	461	490	505	491	416	257
10°	448	464	494	507	490	415	256
15°	448	458	498	509	490	414	256
20°	448	471	502	511	490	412	255
25°	448	474	505	513	490	411	255
30°	448	478	509	516	489	410	254
35°	448	482	513	517	489	408	254
40°	448	486	517	519	488	407	253
45°	448	491	522	521	487	406	252
50°	448	496	526	522	486	404	252
60°	448	509	534	522	482	401	250

Table 2

ρ \ α	0°	15°	30°	45°	60°	75°	85°
0°	633	640	654	660	650	602	446
5°	633	642	656	661	649	600	446
10°	633	645	659	662	649	599	445
15°	633	647	661	663	648	597	444
20°	633	649	663	663	647	596	444
25°	633	651	664	664	646	594	443
30°	633	653	666	664	646	593	443
35°	633	655	668	663	645	592	442
40°	633	658	669	663	643	591	441
45°	633	660	670	662	642	589	441
50°	633	663	670	660	640	588	439
60°	633	668	667	650	632	586	438

Table 3

ρ \ α	0°	15°	30°	45°	60°	75°	85°
0°	718	723	730	731	722	690	557
5°	718	724	732	731	721	689	556
10°	718	726	733	731	720	688	556
15°	718	727	734	731	719	687	556
20°	718	728	735	731	718	686	555
25°	718	729	735	730	717	685	555
30°	718	730	735	729	716	684	554
35°	718	731	735	728	715	683	554
40°	718	732	734	726	713	682	553
45°	718	733	733	723	711	682	553
50°	718	734	731	719	708	682	553
60°	718	735	722	704	699	681	552

apex angle β (Fig. 3). The direction of the acceleration of gravity is opposite to that of the y_1-axis. The slope contains a rectilinear tectonic crack which reaches the slope side face at depth H and angle α, and is filled with low strength rock. The parameter K_{IIc}, which defines the inhomogeneity of the filler and its varying resistance to slip at the head of the slip line that develops along the tectonic discontinuity. The slip line length is denoted by l. The variation of filler strength may be caused, for example, by a difference in the moisture content owing to rain or ground water filtration.

We shall solve the problem by the following method of successive approximations (of the kind of the Mikhlin-Sherman method). In the first approximation the problem of slope wthout a crack is solved; in this approximation all boundary conditions of the input problem are satisfied, except the condition for the force at the slip line $\tau_{nt} = k - \sigma_n \operatorname{tg} \rho$. In the second approximation the problem is solved for the half-plane with a slit along the slip line (see boundary conditions (1.4) — (1.8)) with the "discrepancy" in $f(r)$ determined by the previous approximation.

In the third approximation the problem of slope without crack loaded only at the ground day surface $y_1 = 0$ is solved with the load determined in the second approximation, etc. The stress concentration factor K_{II} is determined by the second, fourth, etc. approximations. Let us restrict the analysis to the second approximation, and assume that the length l is fairly small in comparison with H. The case of $l \sim H$ is almost always close to the critical state, since then considerable tensile stresses appear in the day surface above the end of the slip line so that normal rupture occurs along the perpendicular to the day surface.

First, let us determine the stress field in a homogeneous isotropic heavy slope without a tectonic crack, and assume that in addition to forces of gravity the massif is subjected to the following normal surface loads:

$$\begin{aligned} &\text{for} \quad \theta_1 = 0 \quad \sigma_\theta = -q_0 - \delta_1 x_1, \quad \tau_{r\theta} = 0 \\ &\text{for} \quad \theta_1 = -\beta \quad \sigma_\theta = -q_1 + \delta_0 y_1, \quad \tau_{r\theta} = 0 \end{aligned} \tag{5.1}$$

where δ_0, δ_1, q_0 and q_1 are some positive constants, $r_1\theta_1$ are polar coordinates whose origin lies at the vertex of the angle of slope (Fig. 3). The second boundary condition for $\delta_0 \neq 0$ is important when the slope is a sea- or river-shore.

Stresses σ_r, σ_θ and $\tau_{r\theta}$ in the considered heavy massif can be represented with the use of the Kolosov-Muskhelishvili potentials $\Phi(z)$ and $\Psi(z)$ as follows [4]:

$$\begin{aligned} &\sigma_r + \sigma_\theta = 2\left[\Phi(z) + \overline{\Phi(z)}\right] + \frac{\delta(z - \bar{z})}{2i(1-\nu)} \\ &\sigma_r - \sigma_\theta + 2i\tau_{r\theta} = e^{2i\theta_1}\left[2\bar{z}\Phi'(z) + 2\Psi(z) + \frac{\delta(1-2\nu)}{2i(1-\nu)}(z - \bar{z})\right] \\ &(z = x_1 + iy_1 = r_1 e^{i\theta_1}) \end{aligned} \tag{5.2}$$

where δ is the specific gravity of rock.

We seek a solution of the singular boundary value problem (5.1),(5.2) of the form

$$\Phi(z) = E_1 z + E_3 + E_0 \ln z, \quad \Psi(z) = E_2 z + E_4 + E_5 \ln z \tag{5.3}$$

where $E_0, \ldots, E_5$ are some complex constants. (It can be readily shown that on group considerations this is the only possible form of solution [1]). We add the first and second formulas in (5.2) and obtain the representation for the combination $\sigma_\theta + i\tau_{r\theta}$

specified for $\theta_1 = 0$ and $\theta_1 = -\beta$; by substituting (5.3) into the latter we obtain the following equations:

$$E_0 + E_3 + \bar{E}_3 + E_4 = -q_0, \quad 2E_1 + \bar{E}_1 + E_2 = -\delta_1 \tag{5.4}$$

$$2E_1 e^{-\beta i} + \bar{E}_1 e^{\beta i} + E_2 e^{-3\beta i} = \frac{\delta \sin\beta}{2(1-\nu)}[1 + (1-2\nu)e^{-2\beta i}] - \delta_0 \sin\beta$$

$$E_0 + \bar{E}_0 + E_5 = 0, \quad E_0 + \bar{E}_0 + E_5 e^{-2\beta i} = 0$$

$$\beta i(\bar{E}_0 - E_0) + E_0 - \beta i E_5 e^{2\beta i} + E_3 + \bar{E}_3 + E_4 e^{-2\beta i} = -q_1$$

We write the solution of this system of equations as

$$E_0 = \frac{i(q_0 - q_1)\sin 2\beta}{2(\beta \sin 2\beta - 1 + \cos 2\beta)}, \quad E_5 = 0 \tag{5.5}$$

$$E_3 = \frac{q_1(1-\cos 2\beta) - q_0(2\beta \sin 2\beta - 1 + \cos 2\beta)}{4(\beta \sin 2\beta - 1 + \cos 2\beta)}$$

$$E_4 = -q_0 - \frac{(q_0 + q_1)(1 - \cos 2\beta) - 2\beta q_0 \sin 2\beta + i(q_0 - q_1)\sin 2\beta}{2(\beta \sin 2\beta - 1 + \cos 2\beta)}$$

$$E_1 = \frac{B(\cos 2\beta - 1) + i(B - \bar{B})\sin 2\beta}{8 \sin^4 \beta}$$

$$E_2 = -\delta_1 - \frac{(2B + \bar{B})(\cos 2\beta - 1) + i(B + \bar{B})\sin 2\beta}{8 \sin^4\beta}$$

$$B = \delta_0 \sin\beta e^{\beta i} - \delta_1 e^{-2\beta i} - \frac{\delta \sin\beta}{2(1-\nu)} e^{\beta i}[1 + (1-2\nu)e^{-2\beta i}]$$

The case of local loads applied near the slope angle are conveniently investigated by the method of Belonosov [5]. Using (5.2) we compute in the derived solution (5.3) and (5.5) the normal and shear stresses along the tectonic rupture for

$$z = He^{-\beta i} + r \exp i(\alpha - \beta + \pi/2)$$

where r is the distance from the crack base on the side surface of the slope (Fig. 3). We have

$$\sigma_n + i\tau_{tn} = 2\,\mathrm{Re}\,(E_1 z + E_3) + \frac{\delta(z - \bar{z})}{4i(1-\nu)} + 2\,\mathrm{Re}\,(E_0 \ln z) - \tag{5.6}$$
$$e^{2i(\alpha-\beta)}\left[E_0 \frac{\bar{z}}{z} + E_1 \bar{z} + E_2 z + E_4 + \frac{\delta(1-2\nu)}{4i(1-\nu)}(z - \bar{z})\right] =$$
$$2\,\mathrm{Re}\,E_3 + 2H\,\mathrm{Re}\,(E_1 e^{-i\beta}) - \frac{\delta H \sin\beta}{2(1-\nu)} - e^{2i(\alpha-\beta)}\Big[E_4 +$$
$$E_1 H e^{i\beta} + E_2 H e^{-i\beta} - \frac{\delta H \sin\beta(1-2\nu)}{2(1-\nu)}\Big] + r\Big\{-2\,\mathrm{Im}\,[E_1 e^{i(\alpha-\beta)}] +$$
$$\frac{\delta \cos(\alpha-\beta)}{2(1-\nu)} - e^{2i(\alpha-\beta)}\Big[\frac{\delta(1-2\nu)\cos(\alpha-\beta)}{2(1-\nu)} + E_2 i e^{i(\alpha-\beta)} -$$
$$iE_1 e^{-i(\alpha-\beta)}\Big]\Big\} - 2\arg z\,\mathrm{Im}\,E_0 - E_0 \exp 2i(\alpha - \beta - \arg z)$$

$$\arg z = \mathrm{arctg}\,\frac{-H\sin\beta + r\cos(\alpha-\beta)}{H\cos\beta - r\sin(\alpha-\beta)} \quad (-\beta < \arg z < 0$$

We compute function $f(r)$ by formulas (5.6)

$$f(r) = -(\tau_{tn} + \sigma_n \operatorname{tg} \rho) = -\frac{1}{\cos \rho} \operatorname{Im} [e^{i\rho} (\sigma_n + i\tau_{tn})]$$

which determines the second approximation solution (see Sects. 3 and 4) and also, the stress concentration factor. For convenience of computations we expand the last two non-linear terms in (5.6) in series in r in the vicinity of point $r = {}^1/_2 l$. We restrict the expansion to the first two terms and then use the results of computations carried out in Sect. 4 for the case of the linear function $f(r)$.

Final formulas in dimensionless form are

$$f(r) = a + \eta_1 r \tag{5.7}$$

$$K_{II} = -\frac{\sqrt{\pi l}\,(a+k)}{\sqrt{2}\, G^+(-1)} - \frac{3\eta_1 \sqrt{2\pi}\, l^{3/2}}{8G^+(-2)} \tag{5.8}$$

where

$$a = -\frac{1}{\cos \rho} \operatorname{Im} \left\{ e^{i\rho} \left[2 \operatorname{Re} E_3 + 2H \operatorname{Re} (E_1 e^{-i\beta}) - \right.\right. \tag{5.9}$$

$$\frac{\delta H \sin \beta}{2(1-\nu)} - e^{2i(\alpha-\beta)} \left(E_4 + E_1 H e^{i\beta} + E_2 H e^{-i\beta} - \right.$$

$$\left. \frac{\delta H \sin \beta}{2} \frac{1-2\nu}{1-\nu} \right) - 2 \operatorname{Im} E_0 \left(\Delta - \frac{2Hl \cos \alpha}{4H^2 - 4Hl \sin \alpha + l^2} \right) -$$

$$\left.\left. E_0 \left(1 + \frac{4Hli \cos \alpha}{4H^2 - 4Hl \sin \alpha + l^2} \right) e^{2i(\alpha-\beta-\Delta)} \right] \right\}$$

$$\eta_1 = -\frac{1}{\cos \rho} \operatorname{Im} \left\{ e^{i\rho} \left[-2 \operatorname{Im} (E_1 e^{i(\alpha-\beta)}) + \frac{\delta \cos (\alpha-\beta)}{2(1-\nu)} - \right.\right.$$

$$e^{2i(\alpha-\beta)} \left(\frac{1-2\nu}{1-\nu} \frac{\delta \cos (\alpha-\beta)}{2} + iE_2 e^{i(\alpha-\beta)} - iE_1 e^{-i(\alpha-\beta)} \right) -$$

$$\left.\left. \frac{8H \cos \alpha \operatorname{Im} E_0}{4H^2 - 4Hl \sin \alpha + l^2} + \frac{8iE_0 H \cos \alpha}{4H^2 - 4Hl \sin \alpha + l^2} e^{2i(\alpha-\beta-\Delta)} \right] \right\}$$

$$\Delta = \operatorname{arctg} \frac{-2H \sin \beta + l \cos (\alpha-\beta)}{2H \cos \beta - l \sin (\alpha-\beta)} \quad (-\beta < \Delta < 0)$$

In the limit state the quantity $|K_{II}|$ is equal to the constant K_{IIc}. Using this and (5.8) we obtain the formula

$$\frac{a+k}{G^+(-1)} + \frac{3\eta_1 l}{4G^+(-2)} = -\frac{\sqrt{2}\, K_{IIc}}{\sqrt{\pi l}} \tag{5.10}$$

which relates the slip line length to the applied loads. If $K_{IIc} \ll \sqrt{\pi l}\, |a+k|$, i.e. the maximum admissible stress concentration is small in comparison with the mean stress, then by virtue of (5.10) we have

$$l = -\frac{4(a+k)\, G^+(-2)}{3\eta_1 G^+(-1)} \tag{5.11}$$

This case can be of practical importance for very low strength fillers (e.g. of the kind of moist clay). It should be emphasized that the problem that obtains in this limit case is the classical elasto-plastic problem of the development of thin plastic slip layers (for metals $\rho = 0$ can be assumed in this case). For an inhomogeneous filler parameter K_{IIc} in (5.10) is to be taken as some specified function of l.

Investigation of the slip line motion stability is carried out by methods of fracture mechanics [1]. In the considered problems the necessary and sufficient condition of stability is of the form $\partial K_{II} / \partial l < 0$, i.e. by virtue of (5.8) and (5.10)

$$\frac{a+k}{G^+(-1)} > \frac{-9\eta_1 l}{4G^+(-2)} \tag{5.12}$$

As an illustration of the derived solution we present below the dependence of the dimensionless critical height of the slope $K_{IIc} / (\delta H \sqrt{\pi l})$ on the dimensionless parameters $k / \delta H$ and l / H for a load-free slope for $\nu = 0{,}3$, $q_0 = q_1 = 0$, $\delta_0 = \delta_1 = 0$, $\beta = 120°$, $\alpha = 75°$ and $\rho = 30°$

$$\frac{K_{IIc}}{\delta H \sqrt{\pi l}} = 0.46 - 0.26 \frac{k}{\delta H} - 0.034 \frac{l}{H}$$

The obtained solution can be, evidently, used also for the experimental determination of parameter K_{IIc}, for instance, in experiments on uniaxial compression of specimens with an artificial boundary discontinuity along an inclined bonding plane (formula (5.9) for $\eta_1 = 0$ and a is equal to the right-hand part of equality (1.9)). The properties of the bond along the slip line and its continuation must simulate the properties of the filler in the tectonic crack and its interaction with the basic rock (quantities k and ρ of the adhesive must in any case be equal to the related minimum values of k and ρ that are characteristic for the pairs filler-filler and filler-rock in the limit and the sliding states). The practical difficulties of simulating the structure of the "head" slip line are, evidently, not smaller than in the case of crack of normal cleavage.

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SOME ELASTIC-PLASTIC PROBLEMS FOR A PLANE WEAKENED BY A PERIODIC SYSTEM OF CIRCULAR HOLES

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Elastic-plastic problems for a plane weakened by an infinite series of circular holes are considered. It is assumed that the stress level and the spacing between the holes are such that the circular holes are entirely enclosed by the appropriate